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# **Irrational Numbers**

An **Irrational Number** is a number that **cannot** be written as a simple fraction - the decimal goes on forever without repeating.

Example:  $\pi$  (Pi) is an irrational number. The value of Pi is

#### 3.1415926535897932384626433832795 (and more...)

There is no pattern to the decimals, and you cannot write down a simple fraction that equals Pi.



It is called <u>irrational</u> because it cannot be written as a <u>ratio</u> (or fraction), not because it is crazy! HOWEVER –Much like an irrational person irrational numbers are NOT predictable!

### **Rational vs Irrational**

But if a number **can** be written as a simple fraction then it is called a **rational number**:

Example: **9.5** can be written as a simple fraction like this

$$^{19}/_{2} = 9.5$$

So it is **not** an irrational number (and so is a **rational number**)

Here are some more examples:

Number	As a Fraction	Rational or Irrational?
5	5/1	Rational
1.75	7/4	Rational
.001	1/1000	Rational
$\sqrt{2}$ (square root of 2)	?	Irrational!

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## **Example: Is the Square Root of 2 an Irrational Number?**

My calculator says the square root of 2 is 1.4142135623730950488016887242097, but this is not the full story! It actually goes on and on, with no pattern to the numbers.

You **cannot** write down a simple fraction that equals the square root of 2. So the square root of 2 is an *irrational number* 



When dealing with square roots a lot of them are classified as IRRATIONAL numbers. The special few that are RATIONAL are absolutely PERFECT!

\*\*The square root of any PERFECT SQUARE is RATIONAL!

#### **Famous Irrational Numbers**

π

**Pi** is a famous irrational number. People have calculated Pi to over one million decimal places and still there is no pattern. The first few digits look like this:

3.1415926535897932384626433832795 (and more ...)



The number e (<u>Euler's Number</u>) is another famous irrational number. e is the base of the Natural <u>Logarithms</u> (invented by John Napier). People have also calculated e to lots of decimal places without any pattern showing. The first few digits look like this:

2.7182818284590452353602874713527 (and more ...)



The Golden Ratio is an irrational number (symbol is the Greek letter "phi" shown at left). The first few digits look like this:

1.61803398874989484820... (and more ...)



Many square roots, cube roots, etc are also irrational numbers. Examples:

 $\sqrt{3}$  1.7320508075688772935274463415059 (etc)  $\sqrt{99}$  9.9498743710661995473447982100121 (etc)

But  $\sqrt{4} = 2$ , and  $\sqrt{9} = 3$ , so **not all** roots are irrational.

## **History of Irrational Numbers**

Apparently *Hippasus* (one of *Pythagoras'* students) discovered irrational numbers when trying to represent the square root of 2 as a fraction (using geometry, it is thought). Instead he proved you couldn't write the square root of 2 as a fraction and it was *irrational*.

However *Pythagoras* could not accept the existence of irrational numbers, because he believed that all numbers had perfect values. But he could not disprove *Hippasus'* "irrational numbers" and so Hippasus was thrown overboard and drowned!

(Retrieved on Mathisfun.com)

### **Lets Practice!**

Identify each of the following as rational (R) or irrational (I) by writing the appropriate letter on the blank space provided next to each number:

a. 
$$\sqrt{81}$$

d. 
$$\sqrt{10}$$

b. 
$$\frac{1}{3}$$

f. 
$$\pi$$