

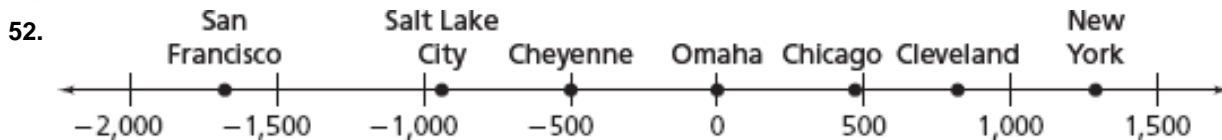
### Applications

1. a.  $-12 + (-4 + 9) = -7$   
 $-12 + 5 = -7$   
 $[-12 + (-4)] + 9 = -7$   
 $-16 + 9 = -7$ 
  - b.  $(14 - 20) - 8 = -14$   
 $-6 - 8 = -14$   
 $14 - (20 - 8) = 2$   
 $14 - 12 = 2$ 

These do not result in the same answer because subtraction is not associative (Associative Property does not hold), while addition is associative.
  - c.  $[14 + (-20)] + (-8) = -14$   
 $-6 + (-8) = -14$   
 $14 + [-20 + (-8)] = -14$   
 $14 + (-28) = -14$
  - d.  $-1 - [-1 + (-1)] = 1$   
 $-1 - (-2) = 1$   
 $[-1 - (-1)] + (-1) = -1$   
 $0 + (-1) = -1$
  - e. Problems with subtraction lead to different results. Subtraction cannot be done in any order. Addition can be done in any order, so changing the order in parts (a) and (c) did not change the answer.
2. 1
3. -5
4. -26
5. 20
6. 6
7. 42
8.  $(-150 + 270) + 30 = (270 + 30) + (-150) = 300 - 150 = 150$ ; addition is commutative and associative.
9.  $(43 \cdot 120) + [43 \cdot (-20)] = 43 \cdot (120 - 20) = 43 \cdot (100) = 4,300$ ; Distributive Property of multiplication (over addition);  $120 + (-20) = 100$
10.  $23 + (-75) + 14 + (-23) - (-75) = [23 + (-23)] + [-75 - (-75)] + 14 = (23 - 23) + (-75 + 75) + 14 = 14$ ; addition is commutative and associative.
11.  $[0.8 \cdot (-23)] + [0.8 \cdot (-7)] = 0.8 \cdot [-23 + (-7)] = 0.8 \cdot (-30) = -24$ ; Distributive Property (of multiplication over addition)
12. a. True. Using the Distributive Property, 432 groups of 50 is the same as 400 groups of 50 plus 32 groups of 50.
  - b. True. Using the Distributive Property,  $400 - 32 = 368$ . 400 groups of 50 minus 32 groups of 50 is the same as 368 groups of 50.
  - c. Not true.  $998 \neq -1000 + 2$
  - d. Not true. The Distributive Property is not applied correctly; multiplication is distributed over addition, not addition distributed over multiplication.
  - e. True. Distributive Property.  $-70 + (-50) = -120$
  - f. True. Distributive Property.  $20 - 3 = 17$
13.  $-2 \cdot [5 + (-8)] = (-2 \cdot 5) + [-2 \cdot (-8)]$
14.  $(-3 \cdot 2) - [-3 \cdot (-12)] = -3 \cdot [2 - (-12)]$
15.  $x \cdot (-3 + 5) = -3x + 5x$
16.  $-7x + 4x = x(-7 + 4)$
17.  $2x \cdot [2 - (-4)] = 2 \cdot 2x - (-4) \cdot 2x$
18.  $x - 3x = x(1 - 3)$
19. No, it does not make a difference.  $0.05 \cdot (7.99 + 3.99) = (0.05 \cdot 7.99) + (0.05 \cdot 3.99)$  because of the Distributive Property.

### Connections

20.  $3 \cdot 12 = 36$
21.  $3 \cdot (-12) = -36$
22.  $-3 \div (-12) = \frac{1}{4}$
23.  $-10 \cdot (-11) = 110$
24.  $-10 + 11 = 1$
25.  $10 - 11 = -1$
26.  $-24 - (-12) = -12$
27.  $\frac{-24}{-12} = 2$
28.  $-18 \div 6 = -3$
29.  $50 \cdot 70 = 3,500$
30.  $50 \cdot (-70) = -3,500$
31.  $2,200 \div (-22) = -100$
32.  $-50 \cdot (-120) = 6,000$
33.  $-139 + 899 = 760$
34.  $5,600 - 7,800 = -2,200$
35.  $-4,400 - (-1,200) = -3,200$
36.  $\frac{-9900}{-99} = 100$
37.  $-580 + (-320) = -900$
38. a. Negative. The opposite of  $3^2$  is  $-(3^2)$  or  $-9$ .  
 b. Negative.  $(-6)^3 = -6 \cdot (-6) \cdot (-6)$  or  $-216$ .  
 c. Positive. A negative number multiplied an even number of times will result in a positive product.  $(-4)^4 = -4 \cdot (-4) \cdot (-4) \cdot (-4)$  or  $256$ .  
 d. Negative.  $1^6 = 1$ , so  $-1^6 = -(1^6)$  or  $-1$ .  
 e. Positive. Four negative numbers multiplied together will result in a positive product.  $-3 \cdot (-3) \cdot (-3) \cdot (-3) = 81$   
 f. Negative. The opposite of  $2^3$  is  $-(2^3)$  or  $-8$ .
39.  $[8 + 4 + 3 + 7 + (-15) + 20 + 5 + (-12) + 32 + 1] \div 10 = 5.3$  yards
40. a.  $-34 + (-15) = -49$   
 b.  $-12 \cdot (-23) = 276$   
 c.  $-532 \div 76 = -7$   
 d.  $-777 - (-37) = -740$   
 e.  $-49 - (-34) = -15$ ;  $-49 - (-15) = -34$   
 f.  $276 \div (-12) = -23$ ;  $276 \div (-23) = -12$
41.  $n - (-5) = 35$ ;  $n = 35 + (-5)$ ;  $5 = 35 - n$ ; answers will vary for the related fact.  $n = 30$ . Some students find the "missing addend" problem to be easier to reason, but other students may find one of the other forms easier, such as  $n = 35 + (-5)$ .
42.  $4 + n = -43$ ;  $n = -43 - 4$ ;  
 $4 = -43 - n$ ;  $n = -47$
43.  $-2n = -16$ ;  $n = -16 \cdot (-\frac{1}{2})$ ;  $n = 8$
44.  $\frac{n}{4} = -32$ ;  $n = -32 \cdot 4$ ;  $n = -128$
45. a.  $[1 + (-3)] \cdot (-4) = 8$   
 b.  $1 + [-3 \cdot (-4)] = 13$   
 c.  $-6 \div [-2 + (-4)] = 1$   
 d.  $[-6 \div (-2)] + (-4) = -1$   
 e.  $[-4 \cdot 2] - 10 = -18$   
 f.  $-4 \cdot (2 - 10) = 32$
46. D
47. A: 0.05, 0.5, 3.140, 14.2, 31.4, 55, 75  
 B:  $\frac{2}{5}, \frac{3}{5}, \frac{9}{8}, \frac{8}{7}, \frac{3}{2}, \frac{5}{3}$   
 C: 0.2, 0.5, 0.6, 0.75, 1, 1.5
48. No.  $|-2 + 3| = 1$ ,  $|-2| + |3| = 5$
49. No.  $5 - |-2 + 3| = 4$ ,  $5 - |-2| + |3| = 6$
50. Yes.  $|-2 - 3| = 5$ ,  $|-2| + |-3| = 5$
51.  $4 \cdot 5 = (4 \cdot 3) + (4 \cdot 2)$



- a. 350 miles east of Omaha or 120 miles west of Chicago;  $50 \text{ mi/h} \cdot 7 \text{ h} = 350 \text{ mi}$
- b. 225 miles west of Omaha or 275 miles east of Cheyenne;  $-50 \text{ mi/h} \cdot 4.5 \text{ h} = -225 \text{ mi}$
- c. 600 miles west of Omaha or 100 miles west of Cheyenne;  $50 \text{ mi/h} \cdot (-12 \text{ h}) = -600 \text{ mi}$
- d. 550 miles east of Omaha or 80 miles east of Chicago;  $-50 \text{ mi/h} \cdot (-11 \text{ h}) = 550 \text{ mi}$

### Extensions

53.  $-23 > -45$

54.  $-23 + 10 > -45 + 10$

55.  $-23 - 10 > -45 - 10$

56.  $-23 \cdot 10 > -45 \cdot 10$

57.  $-23 \cdot (-10) < -45 \cdot (-10)$

58.  $a + c > b + c$

59.  $a - c > b - c$

60.  $a \cdot c > b \cdot c$  if  $c$  is positive

$a \cdot c < b \cdot c$  if  $c$  is negative

$a \cdot c = b \cdot c$  if  $c = 0$

Examples:

$4 > 2$

$2 > -1$

$4 + 3 > 2 + 3$

$2 + 1 > -1 + 1$

$4 - 3 > 2 - 3$

$2 - 1 > -1 - 1$

$4 \cdot 3 > 2 \cdot 3$

$2 \cdot 1 > -1 \cdot 1$

$4 \cdot (-3) < 2 \cdot (-3)$

$2 \cdot (-1) < -1 \cdot (-1)$

61.  $n = -12$

62.  $n = -4$

63.  $n = -4$

64. a.  $12 \div (-8 + 4) = -3$ ;  
 $[12 \div (-8)] + (12 \div 4) = 1.5$

b.  $-12 \div [-5 - (-3)] = 6$ ;  
 $[-12 \div (-5)] - [-12 \div (-3)] = -1.6$

c.  $-2$

d.  $\frac{11}{3}$

e. Division does not distribute over addition or subtraction. When the dividend is a sum or difference of two numbers and the divisor is a rational number, the Distributive Property works. However, when the dividend is a rational number and the divisor is a sum or difference of two numbers, then the Distributive Property does not work.

65. a. The addition is commutative, but the division is not. For example:  $(10 + 15) \div 2 = 12.5$ ;  
 $(15 + 10) \div 2 = 12.5$ ;  
 $(6 + 20) \div 2 = 13$ ;  $(20 + 6) \div 2 = 13$ ;  
addition is commutative, but you must do the addition first, and then divide by 2. You cannot do  $2 \div (6 + 20)$ .

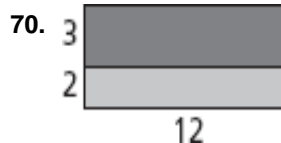
b. Yes, multiplication does distribute over the averaging operation. For example, let  $a = 2$ ,  $x = 4$ , and  $y = 6$ ;  
 $2 (4 \text{ ave } 6) = 2 [(4 + 6) \div 2] = 2 (5) = 10$ ;  
 $(2 \cdot 4) \text{ ave } (2 \cdot 6) = 8 \text{ ave } 12 =$   
 $(8 + 12) \div 2 = 20 \div 2 = 10$ .  $10 = 10$ .

66.  $(5 + 2) \cdot 15$  or  $(5 \cdot 15) + (2 \cdot 15)$

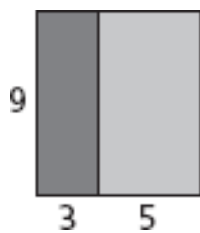
67.  $6 \cdot (12 + 8)$  or  $(6 \cdot 12) + (6 \cdot 8)$

68.  $(5 + 2) \cdot (13 + 2)$  or  
 $(5 \cdot 13) + (5 \cdot 2) + (2 \cdot 13) + (2 \cdot 2)$

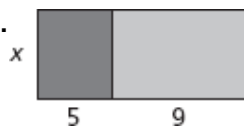
69.  $(x + 2) \cdot 14$  or  $(x \cdot 14) + (2 \cdot 14)$



71.



72.



73.

