Applications

1. **a.** P(green) = $\frac{1}{4}$, P(yellow) = $\frac{2}{4}$, or $\frac{1}{2}$, P(red) = $\frac{1}{4}$

   b. $\frac{1}{4}+\frac{1}{2}+\frac{1}{4}=1$

   c. $\frac{3}{4}$; three of the four blocks are not red.

   d. $\frac{1}{4}+\frac{3}{4}=1$

2. **a.** P(green) = $\frac{12}{25}$, P(purple) = $\frac{6}{25}$, P(orange) = $\frac{2}{25}$, P(yellow) = $\frac{5}{25}$, or $\frac{1}{5}$

   b. $\frac{12}{25}+\frac{6}{25}+\frac{2}{25}+\frac{5}{25}=1$

   c. P(green) = 48%, P(purple) = 24%, P(orange) = 8%, P(yellow) = 20%

   d. 48% + 24% + 8% + 20% = 100%

   e. 1 or 100%; possible explanation: If the possible outcomes of an action do not overlap and account for everything that might happen, then the sum of the probabilities of the outcomes will be 1, or 100%. For example, on the toss of a number cube the outcomes are 1, 2, 3, 4, 5, or 6. If you find the probabilities of an even number or an odd number, you account for everything that might happen. However, if you find the probabilities of an even number or a prime number there is overlap, because 2 is even and prime.

3. **a.** Answers will vary. Students should think these predictions are reasonable, since for Bailey the probability of getting blue is almost 50% and for Jarod the probability of getting red is almost 50%. Both of their predictions had the greatest probability of occurring.

   b. Neither boy did anything wrong in his calculations. Bailey based his on an experiment and Jarod found the theoretical probabilities. While these probabilities are close, it is not likely they will be exactly the same.

4. **a.** P(white) = $\frac{2}{6}$, or $\frac{1}{3}$; P(red) = $\frac{1}{6}$; P(purple) $=\frac{3}{6}$, or $\frac{1}{2}$

   b. $\frac{2}{3}$; the probability of choosing a white block is $\frac{1}{3}$, so the probability of not choosing a white block is $1-\frac{1}{3}=\frac{2}{3}$.

   c. The probabilities are the same. There will be 4 white blocks, 2 red blocks, and 6 purple blocks, so P(white) $=\frac{4}{12}=\frac{1}{3}$; P(red) $=\frac{2}{12}$, or $\frac{1}{6}$; P(purple) $=\frac{6}{12}$, or $\frac{1}{2}$

   d. P(white) $=\frac{4}{12}$, or $\frac{1}{3}$; P(red) $=\frac{3}{12}$, or $\frac{1}{4}$; P(purple) $=\frac{5}{12}$

   e. Answers may vary. Possible answer: Add 4 red blocks. Then there are 5 red blocks, 2 white blocks, and 3 purple blocks, for a total of 10 blocks. And P(red) $=\frac{5}{10}$, or $\frac{1}{2}$.

5. **a.** $\frac{3}{3}=1$

   b. 0

   c. $\frac{0}{3}=0$

6. **a.** Since the probability of choosing a white marble is $\frac{3}{10}$, there must be at least 10 marbles in the bag.

   b. Yes. Possible answer: Let's assume the bag contains 60 marbles. Since the probability of choosing red is $\frac{1}{5}$, then $\frac{1}{5}$ of the proposed 60 marbles, or 12 marbles, would be red. Since $\frac{3}{10}$ of the 60 marbles must be white, 18 would be white. This gives us a total of 30 marbles, so the other 30 marbles would be blue. Since we get whole numbers of marbles when we do this division, the bag could contain 60 marbles.

   c. Possible answer: Since $\frac{3}{10}$ of the marbles in the bag are white, and 6 marbles are white, we need to answer this question: $\frac{3}{10}$ of what number equals 6? The answer is 20. This works because P(red) $\frac{1}{5}$, and $\frac{1}{5}$ of 20 = 4 red marbles. Since 4 marbles are red and 6 are white, there are 20 – 10 = 10 blue marbles in the bag.

d. Possible answer: You could add the other two probabilities (of red and white) and subtract the result from 1: $\frac{1}{5}+\frac{3}{10}=\frac{2}{10}+\frac{3}{10}=\frac{5}{10}$, and $1-\frac{5}{10}=\frac{5}{10}$, or $\frac{1}{2}$. So the probability of choosing a blue marble is $\frac{1}{2}$.

7. a. True. The outcome must be impossible (such as rolling a 7 on a number cube).

b. True. The outcome must be absolutely certain (such as rolling a number less than 7 on a number cube).

c. False. All probabilities are between 0 (impossible) and 1 (absolutely certain), inclusive.

8. The outcomes are the same for the two situations. We can see this by identifying the three coins in the first case with the three tosses in the second. Coin 1 can be heads or tails, just as the first toss can be, and so on. Tossing a coin three times or tossing three coins at once does have the same number of equally likely outcomes. The outcomes include HHH, TTT, THT, HTH, TTH, HHT, THH, and HTT. **Note:** Some students may answer no for this question, which is fine as long as their reasoning is correct. They may say that the outcomes for tossing three coins at once are three heads, three tails, two heads and one tail, or two tails and one head. These descriptions do describe the outcomes, though notice that these outcomes are not equally likely. This must be the case when determining theoretical probability. If the order of heads and tails matters in the game, then tossing one coin 3 times makes it easier to keep track of this. That is, if HTT is considered different from THT in their game, then tossing one coin 3 times will make spotting the difference easier.

9. This is not a fair game. There are two winning outcomes for Eva (THT and HTH), but four winning outcomes for Pietro (TTH, HHT, THH, and HTT). Two outcomes (HHH and TTT) have no winners. The game can be made fair by changing the point scheme. Eva can be awarded twice as many points as Pietro for each winning combination.

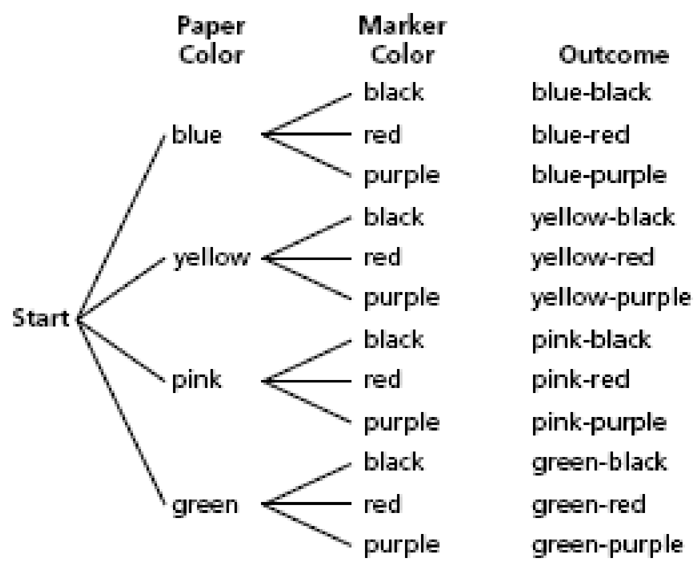
10. a. odd, odd; odd, even; even, odd; even, even

b. Possible answer: If the sum of the two number cubes is even, Player 1 scores a point. If the sum of the two number cubes is odd, Player 2 scores a point.

c. Possible answer: If the product of the two number cubes is even, Player 1 scores a point. If the product of the two number cubes is odd, Player 2 scores a point.

d. As with tossing a coin, considering even and odd on a number cube gives two equally likely outcomes. The game suggested in part (b) is like the game in Problem 1.3, One More Try. In this game, a match is two evens or two odds, which has a sum that is even. No match is one of each, which has an odd sum. The games that students might imagine for part (b) might not be as easily connected to the two-coins game.

11. a. For paper color followed by marker color, there are 12 outcomes.



b. $P(\text{pink, red}) = \frac{1}{12}$

c. $P(\text{blue paper}) = \frac{1}{4}$;

$P(\text{not blue paper}) = \frac{3}{4}$, or $1 - P(\text{blue}) =$

$1 - \frac{1}{4} = \frac{3}{4}$

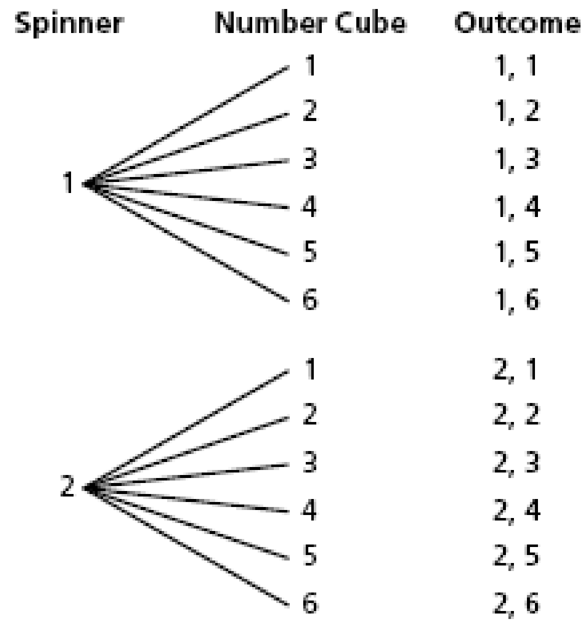
d. $P(\text{purple marker}) = \frac{4}{12}$, or $\frac{1}{3}$

12. a. There are 12 different possible lunches. (See Figure 1.)

b. The probability of Sol getting his favorite lunch is $\frac{1}{12}$. Since the cook is not paying any attention to how she puts the lunches together, and there are equal numbers of all kinds of sandwich, vegetable, and fruit, all of the 12 combinations are equally likely.

c. The probability of Sol getting at least one of his favorite things is $\frac{10}{12}$, or $\frac{5}{6}$. Only 2 of the 12 combinations of items, hamburger-spinach-apple and turkey-spinach-apple don’t contain at least one of his favorite things.

13. a.

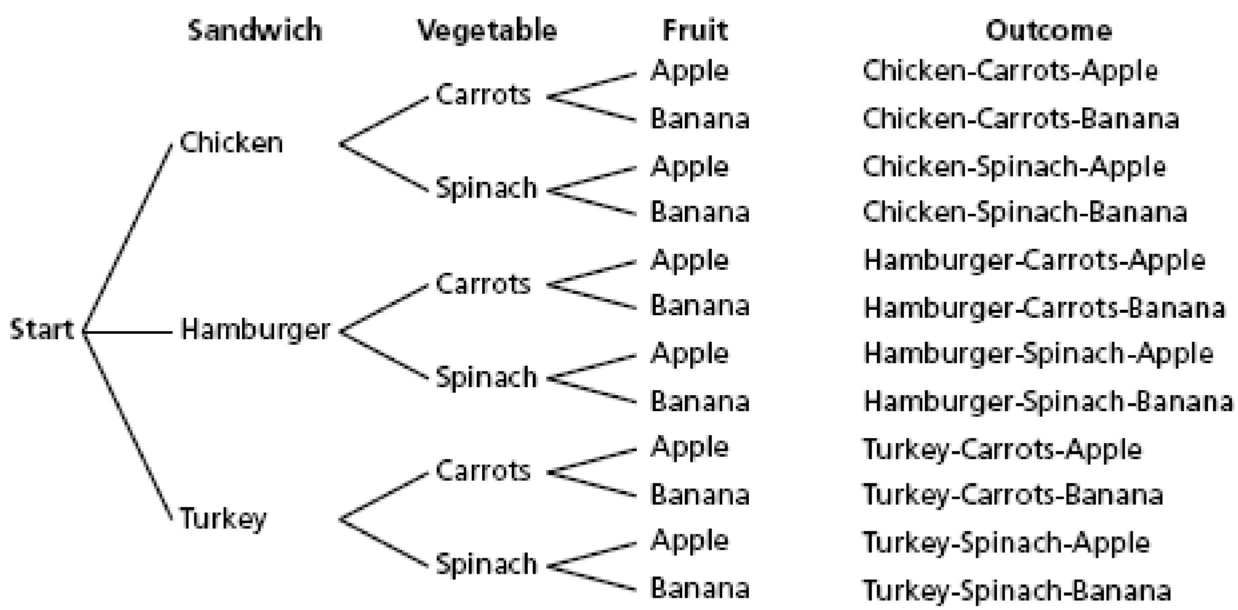


The 12 possible outcomes of a spin followed by a roll of the number cube are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6).

b. Since (2, 2) is one of 12 equally likely possibilities, the probability is $\frac{1}{12}$.

Figure 1



c. Since the factors of 2 are 1 and 2, the only possibilities are (1, 1), (1, 2), (2, 1), and (2, 2). Thus there are 4 ways out of 12 equally likely outcomes, so the probability is $\frac{4}{12}$, or $\frac{1}{3}$.

d. The multiples of 2 in the data are 2, 4, and 6, so the only possibilities are (2, 2), (2, 4), and (2, 6). Thus there are 3 ways out of 12 equally likely outcomes, so the probability is $\frac{3}{12}$, or $\frac{1}{4}$.

Connections

14. **a.** $\frac{1}{8} = \frac{4}{32} = \frac{5}{40}$

b. $\frac{3}{7} = \frac{9}{21} = \frac{6}{14}$

c. $\frac{8}{20} = \frac{2}{5} = \frac{16}{40}$

15. Parts (a) and (b) are both equal to 1.

16. Possible answer: A bag has 12 marbles. One marble is red, four marbles are green, and seven marbles are yellow. Find *P*(red or green or yellow).

17. **a.** Bly's probability is closer to $\frac{1}{3}$.

If students convert the fractions into decimal form, they will find that Bly's probability is 0.3375 and Kara's probability is about 0.417.

b. Possible answers: Tossing a number cube and finding the probability that you will roll a number greater than 4. Choosing a red block from a bag containing one red, one blue, and one green block.

18–25. Answers will vary. Students' answers should be fractions between and including 0 and 1 (or percents between and including 0% and 100%), and their reasoning should justify their answer.

26. Game 1: possible, equally likely, fair
Game 2: possible, unlikely, unfair
Game 3: possible, equally likely, fair
Game 4: possible, unlikely, unfair
Game 5: not possible, unlikely, unfair

27. **a.** 11

b. 66

c. Yes; students might make a tree to show all the possibilities. Some examples: 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, and 32.

d. Karen wins on 11, 13, 23, 31, 41, 43, 53, and 61. Mia wins on 12, 16, 24, 32, 36, 44, 52, 56, and 64. So Mia has a greater chance of winning than Karen. Students can do this experimentally, but the theoretical probabilities are so close that it would be hard for them to design an experiment with a large enough number of trials to arrive confidently at the correct conclusion.

28. A

29. H

30. B

31. G

32. **a.** From counting, we know there are 28 students in the class. Since choosing any student is as likely as choosing another, and 4 of the 28 have first names that begin with J, the probability is $\frac{4}{28}$, or $\frac{1}{7}$.

b. There are 17 names that begin with a letter from G through S, so the probability of choosing a student in this range is $\frac{17}{28}$.

c. $\frac{1}{28}$, because there is only one person in the class whose first name begins with K

d. The class now has 30 students, and since there are still only 4 students whose names begin with J, the new probability is $\frac{4}{30} = \frac{2}{15}$.

33. a. You need to find $\frac{1}{2} \times \frac{1}{7}$ to figure the probability of white. You need to find $\frac{1}{2} + \frac{1}{7} + \frac{1}{14}$ and subtract from 1 to figure the probability of blue.

b. The total number of marbles has to be a multiple of 2, 7, and 14 since these are the denominators of the fractions that give the probabilities.

c. No. The probability of green is $\frac{1}{2}$. There would have to be $3\frac{1}{2}$ green marbles in the bag. There can be any multiple of 14 marbles in the bag.

34. a. $\frac{1}{14}$

b. $\frac{10}{14} = \frac{5}{7}$

Extensions

35. Answers will vary. If students conduct enough trials, the two types of probability should be close.

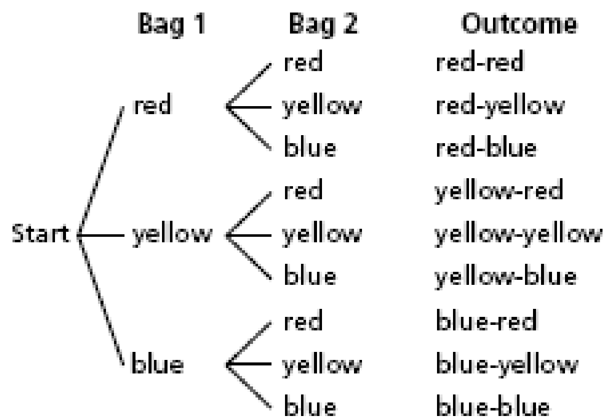
36. a. HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, TTHH, THTH, TTTH, TTHT, THTT, HTTT, TTTT

b. $\frac{1}{16}$

c. Answers will vary. Possible answer: Two players toss the 4 coins and count the number of heads. The player with the most heads wins. If the players tie, then they toss again until one player wins.

37. Contestants should think twice before playing this game. A contestant wins by choosing the combination RR, YY, or BB, so P(match) = $\frac{3}{9}$, or $\frac{1}{3}$, and P(no match) = $1 - \frac{1}{3} = \frac{2}{3}$. Students may argue that having a 1 in 3 chance of winning $5,000 is worth the risk of losing the prizes won so far.

38. a.



b. $\frac{4}{9}$

c. $\frac{1}{4}$

d. No, Jason didn't list all the outcomes for Bag 1. He only has two outcomes, which are blue and not blue, and these are not equally likely outcomes. He should have blue, red, and white under Bag 1 and for each outcome starting under Bag 1. Under Bag 2 he should also have three branches, one for blue, one for red, and one for white. **Note:** Make sure that students notice that the outcomes for each action in the tree, in this case choosing from Bag 1 and choosing from Bag 2, must have all the possible outcomes accounted for.